The source for advantage in noise quantum metrology

Kaonan Micadei,[1,*] Kavan Modi,[2,3,†] Lucas C. Céleri,[1,‡] and Roberto M. Serra[1,§]

[1] *Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, Rua Santa Adélia 166, 09210-170, Santo André, São Paulo, Brazil*

[2] *Department of Physics, University of Oxford, Clarendon Laboratory, Oxford, OX1 3PU, United Kingdom*

[3] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore*

Quantum mechanical systems can be used to out perform classical ones in several tasks. For instance, quantum correlations can be employed to beat the shot-noise limit in metrology protocols. Such parameter estimation methods are crucial for both advances in science and the development of technologies. Almost all quantum technologies operate with some level of noise and how quantum-enhancement fares in the presence of noise is still unclear. Here we show that, when a system in mixed multipartite quantum state is used to estimate a parameter, the information must be decoded by coherent interactions between the parts of the probe. Strangely, we find that the quantum Fisher information is adaptively additive for any pure entangled states, that is, no coherent processing is necessary. This leads to an operational interpretation for the ability of performing coherent interactions in parameter estimation and highlights a fundamental difference between mixed and pure states. This result has fundamental importance in the search for the source of quantum advantage and it also has practical relevance for the designing of new high-precision-measurement devices.

A general framework to estimate a parameter involves a suitable probe and an interaction that manifests the parameter physically. The probe, initially in state ϱ, acquires some information about the parameter, ϕ, encoded in state ϱ_ϕ, which is then read out by some convenient strategy. The estimation process depends on how much information is carried by the encoded probe. The precision of the estimation protocol is limited by Cramér-Rao bound [1, 2], in which the root mean square error, $\Delta\phi$, is bounded by the Fisher information [3], $\mathcal{F}_\phi$, as

$$\Delta\phi \geq \frac{1}{\sqrt{\mathcal{F}_\phi}}. \tag{1}$$

Fisher information is a key concept in metrology and gives us information about the effectiveness of a parameter estimation protocol, i.e., it quantifies how effective is the information codification in ϱ_ϕ. The probe read out is performed in a measurement sampling, resulting in the probability distribution $p(\varrho_\phi)$, which contains the information about ϕ. Inequality (1) may be saturated in a large number of trials. The preceding statements are general statements of estimation theory and, therefore, are true for both quantum and classical systems.

An essential ingredient in parameter estimation is using correlated states of the probes. A correlated probe can be used to improve the estimation process. For the quantum case, non-classical correlations offer considerable advantages in quantum metrology [5, 6] even in noise scenarios [7]. Quantum Fisher information is defined as the supremum over all possible measurements [8]. This raises the important question about whether the final measurement have to be classical or quantum. In this Letter, we address the role of coherent interactions in noise quantum metrology. We show that such operations play a crucial and important role in noise reduction, which leads to a gap between the classical and quantum scenarios. By quantum we mean a global measurement and by classical we mean a local adaptive measurement scheme.

Classical version. Let us consider the following game between three characters: Alice, Bob, and Gregory. The characters receive the same encoded probe ϱ_ϕ. We will consider a bipartite probe for simplicity, but all the discussions we drawn can be easily generalised for the multipartite case. Gregory has access to the whole sampling, while Alice and Bob have access only to local partitions of the encoded probe. The classical scenario for the game is sketched in Figs. 1(a) and (b). We suppose that Gregory has an apparatus able to preform joint measurements on the global sampling involving both variables $\{a, b\}$ of a bipartite probability distribution $p_\phi(a, b)$ (see Fig. 1 (a)). The amount of information that Gregory could learn about ϕ is bounded by Fisher information

$$\mathcal{F}(A, B) = \int\int \mathrm{d}a\mathrm{d}b\, p_\phi(a, b) \left[\frac{1}{p_\phi(a, b)} \frac{\partial}{\partial\phi} p_\phi(a, b)\right]^2. \tag{2}$$

Now, supposing that each partition of the probe is distributed among the two partners, such that the probe variable a is given to Alice, while the probe variable b is assorted to Bob. Both partners are allowed to perform local measurements and operations on their own partitions and they can communicate to each other. At this point, Gregory puts up a challenge to Alice and Bob. The two partners (Alice and Bob) must to estimate the value of ϕ with the same uncertainty that Gregory gets from global measurements. Alice and Bob can employ a local adaptive strategy, depicted in Fig. 1(b), to perform such a challenge task. First, Bob performs a suitable measurement on the variable b and communicates his result to Alice. Next, Alice performs a measurement on

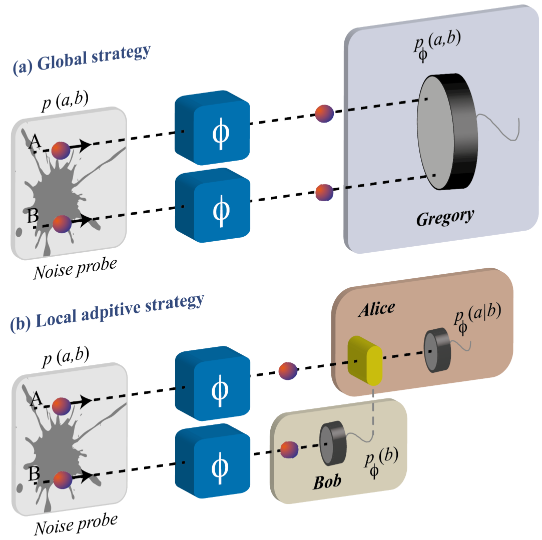

FIG. 1. Sketch of two classically equivalent strategies for parameter estimation. (a) the global and (b) the local adaptive.

the variable a in an adaptive way by employing Bob's result. In this local strategy Bob can learn an amount of information about ϕ that is bounded by Bob's Fisher information

$$\mathcal{F}(B) = \int \mathrm{d}b\, p_\phi(b) \left[\frac{1}{p_\phi(b)} \frac{\partial}{\partial \phi} p_\phi(b)\right]^2, \tag{3}$$

where $p_\phi(b) = \int \mathrm{d}a\, p_\phi(a,b)$. On the other hand, Alice can learn an amount of information associated to a conditional read out bounded by the conditional Fisher information [for the conditional distribution $p_\phi(a|b)$], which we define as

$$\mathcal{F}(A|B) \equiv \int \mathrm{d}b\, p_\phi(b) \mathcal{F}(A|B=b), \tag{4}$$

with $\mathcal{F}(A|B=b) \equiv \int \mathrm{d}a\, p_\phi(a|b) \left[\frac{1}{p_\phi(a|b)} \frac{\partial}{\partial\phi} p_\phi(a|b)\right]^2$.

Comparing Gregory's, Bob's, and Alice's Fisher information, we conclude that, in the classical scenario, Alice and Bob together win the game. By using a local adaptive strategy, Alice and Bob are able to estimate the value of the parameter ϕ reaching the same precision achieved by Gregory. The classical equivalence of the two strategies (global and local adaptive) lies in Bayes' rule. Since the joint probability can be written as $p_\phi(a,b) = p_\phi(b) p_\phi(a|b)$, we can show the following relation between local, conditional and global Fisher informations:

$$\mathcal{F}_C(A,B) = \mathcal{F}_C(B) + \mathcal{F}_C(A|B). \tag{5}$$

The left hand side of Eq. (5) is the Fisher information associated with a global sampling strategy (Fig. 1(a)), whereas the right hand side can be understood as the Fisher information of a local adaptive strategy as depicted in Fig. 1(b). Above we have used the subscript C to denote that Eq. (5) holds in the classical case.

Quantum version. In order to move the challenge game to the quantum word, we will employ the phase estimation protocol depicted in Figs. 2(a) and (b) where the bipartite probe is composed by a two-qubit mixed state, ρ_0^{AB}. Here, the challenge game is to estimate the phase shift ϕ encoded in the probe through the evolution operator $U(\phi) = e^{-i\phi\mathcal{H}^{AB}}$, with the Hamiltonian $\mathcal{H}^{AB} = \mathcal{H}^A \otimes \mathbb{1}^B + \mathbb{1}^A \otimes \mathcal{H}^B$. Gregory's global strategy consists of performing a joint measurement on the product state space $\xi_A \otimes \xi_B$. Performing such a joint measurement in general requires using a entangling operation —the second controlled-not (C-NOT) gate in Fig. 2(a)— followed by local measurements. In other words, the entangling operation involves a coherent interaction between the two partitions. Therefore, in the quantum version of the challenge game, Gregory is able to perform coherent interactions between two partitions of the encoded probe state. To inspect how much can Gregory learn about the phase ϕ, we use the quantum version of the Fisher information [8]. For the probe-system interaction described by Hamiltonian $\mathcal{H}^{AB}$, Gregory's quantum Fisher information is given by

$$\mathcal{F}_G(\rho_\phi^{AB}) = 4 \sum_{i<j} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle \lambda_i | \mathcal{H}^{AB} | \lambda_j \rangle|^2, \tag{6}$$

where $\{\lambda_i\}$ and $\{|\lambda_i\rangle\}$ are the eigenvalues and the eigenstates of the initial probe state ρ_0^{AB}.

In the quantum version of the local adaptive strategy, Alice and Bob are not able to perform a disentangling gate (or a coherent interaction). They only have access to local measurements concatenated by classical communications. Bob can perform a general projective measurement represented by the complete set of projectors $\{\Pi_b^B\}$. After the measurement, Bob communicates his measurement outcome to Alice, which can perform a classical controlled unitary and another local measurement. The estimation process can be optimized by a suitable choice of Bob's measurement basis and Alice's unitary transformation. In this context, we can also define a quantum conditional Fisher information as

$$\mathcal{F}(\rho^{A|B}) \equiv \sum_b p_b \mathcal{F}(\rho^{A|b}), \tag{7}$$

where $\rho^{A|b} = \mathrm{Tr_B}(\mathbb{1}^A \otimes \Pi_b^B\, \rho^{AB} \mathbb{1}^A \otimes \Pi_b^B)/p_b$ is the post-measurement state associated to the b-th outcome, which occurs with probability $p_b = \mathrm{Tr_{AB}}(\mathbb{1}^A \otimes \Pi_b^B\, \rho^{AB})$.

From the classical scenario in Eq. (5), we have $\mathcal{F}_{AB}(\rho^{AB}) = \mathcal{F}(\rho^{A|B}) + \mathcal{F}(\rho^B)$. We define the difference

$$\begin{aligned}\Delta\mathcal{F} \equiv& \mathcal{F}_G(\rho^{AB}) - \mathcal{F}_{AB}(\rho^{AB}) \\ =& \mathcal{F}_G(\rho^{AB}) - \max_{\{\Pi_j^B\}} \left\{ \mathcal{F}(\rho^B) + \sum_b p_b \mathcal{F}(\rho^{A|b}) \right\}, \end{aligned} \tag{8}$$

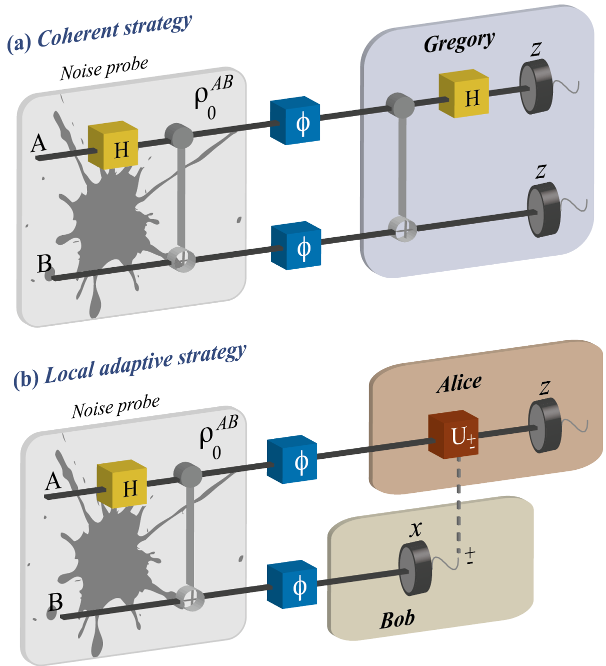

FIG. 2. Quantum algorithms for two different strategies for phase estimation. (a) Global sampling strategy, in this protocol Gregory can perform coherent interactions and he can correlates the sample data to improve the estimation. (b) Local adaptive strategy, in this scenario there is no access to coherent interactions or joint measurements. Bob performs a local measurement in the σ_x^B basis ($\{|+\rangle_B, |-\rangle_B\}$ with $|\pm\rangle_B = (|0\rangle_B \pm |1\rangle_B)/\sqrt{2}$). Conditioned to Bob's outcome, Alice applies a unitary $\{U_+ = H^A, U_- = \sigma_z^A H^A\}$, where σ_j are the usual j-th component of the Pauli matrix and H^A is the Hadamard gate.

as the *discordant Fisher information*, since the aforementioned expressions for the conditional Fisher information may not be equivalent in a general quantum context. In the classical scenario $\Delta\mathcal{F} = 0$. Differently from the classical perspective, a measurement in quantum mechanics disturbs the system state. Moreover the global strategy encompass coherent interactions between the probe partitions. The quantity $\Delta\mathcal{F}$ measures the classical-quantum non-equivalence regarding the difference between a global and a local adaptive strategy, as depicted in Figs. 2(a) and (b).

There are several characteristic traits of quantum mechanics that distinguish it from the classical theory. Besides non-classical correlations like entanglement and discord (which may be present even in separable states) [10, 11], the possibility of performing coherent interactions between different partitions of the probe does not have a classical analogue [12]. This kind of operation implies in the possibility of path interference, which is absent in the standard classical statistics. The role of the coherent interaction in the quantum advantage is a topical issue under recent investigation. Until now we just have few hints about the role played by this important ingredient in the quantum-advantage recipe [13]. The discordant Fisher information defined in Eq. (8) gives us a way to test the role played by coherent interactions in quantum metrology and related tasks. To get a grasp of this, we will explore some different classes of probe states in the aforementioned challenge game.

Pure states. Remarkably, for pure states the global and the local adaptive strategies can reach the same precision for parameter estimation (see [4] for the proof). Therefore, the discordant Fisher information vanishes for any probe in a pure state, independent of the number of parties involved and whether they are entangled or not. In this case, Alice and Bob win the challenge and the coherent interaction in the read-out stage of Gregory's protocol does not play an essential role.

Nevertheless, in the real word applications, it is almost impossible to get rid of all kind of noise. So, in practical situations (in the laboratory), we always have a probe with some level of noise. We will show that for a probe in a mixed state, $\Delta\mathcal{F}$ could be greater than zero (and Gregory could win the game), revealing a gap between classical and quantum estimation theory. Such a gap also reveals a fundamental difference between the pure and the mixed state context. Supposing that the probe preparation depicted in Fig. 2 (a) and (b) has some added noise, we will compute $\Delta\mathcal{F}$.

Mixed states. Let us choose the probe as a Werner state $\rho_0^{AB} = \frac{\eta}{4}\mathbb{1}^{AB} + (1-\eta)\,|\psi^+\rangle_{AB}\langle\psi^+|$, with $\eta \in [0,1]$ and $|\psi^\pm\rangle_{AB} = (|00\rangle_{AB} \pm |11\rangle_{AB})/\sqrt{2}$. The parameter η indicates the amount of white noise present in the probe state. For $\eta < 2/3$ state ρ_0^{AB} is entangled and for $2/3 \leq \eta < 1$ it is separable exhibiting a non-classical correlation revealed by quantum discord [11]. We will consider the phase shift introduced by the unitary $U = \exp\left[i\phi\left(|1\rangle_A\langle 1| \otimes \mathbb{1}^B + \mathbb{1}^A \otimes |1\rangle_B\langle 1|\right)\right]$ following the protocols depicted in Figs. 2 (a) and (b). The eigenvectors of ρ_0^{AB} are $\{|\psi^+\rangle, |\psi^-\rangle, |01\rangle, |10\rangle\}$, with eigenvalues $\{\frac{4-3\eta}{4}, \frac{\eta}{4}, \frac{\eta}{4}, \frac{\eta}{4}\}$, respectively.

The Fisher information of the Gregory's strategy is $\mathcal{F}_G(\rho^{AB}) = 8\frac{(1-\eta)^2}{2-\eta}$. While, for local adaptive strategy, the isolated Bob's Fisher information is null, since he only gets random bits, communicating his measurements outcomes to Alice. Therefore $\Delta\mathcal{F} = \mathcal{F}(\rho^{AB}) - \max_{\{\Pi_j^B\}}\sum_j p_j\mathcal{F}(\rho^{A|j})$. Furthermore, any positive operator values measurement (POVM) can be fine grained to be rank-one, which then always results in

$$\rho^{A|b=\pm} = \frac{1}{2}\begin{bmatrix} 1 & \pm e^{-i2\phi}(1-\eta) \\ \pm e^{i2\phi}(1-\eta) & 1 \end{bmatrix}, \qquad (9)$$

for Alice's conditional state due to Bob's measurement. Therefore, Bob's choice of measurement that optimize the read-out process, is $\{|+\rangle\langle+|, |-\rangle\langle-|\}$, where $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. Thus, the conditional Fisher information

is $\mathcal{F}(\rho^{A|B}) = 4(1-\eta)^2$. Right away we have

$$\Delta\mathcal{F} = (1-\eta)^2\left(\frac{4\eta}{2-\eta}\right), \tag{10}$$

which is non-vanishing for all values for $\eta \neq 0, 1$ and Gregory wins the challenge.

In this example, Gregory's ability to perform coherent interactions plays a non-trivial role in phase estimation in the presence of noise. Revealing a sharper gap between pure and mixed state metrology, as well as, between the classical and the quantum scenarios. In the quantum scenario, coherent interactions can be employed to reduce the noise of the probe state, making more effective Gregory's strategy for phase estimation.

Gregory's advantage can be better understood by looking to the two different read-out strategies. After Gregory performs the second C-NOT (depicted in Fig. 2(a)), the encoded probe state turns out to be

$$\rho_\phi^{AB} \to \left(\frac{1-\eta}{2}\left|\varphi\right\rangle_A\left\langle\varphi\right| + \frac{\eta}{4}\mathbb{1}\right)\otimes\left|0\right\rangle^B\left\langle 0\right| + \frac{\eta}{4}\mathbb{1}^A\otimes\left|1\right\rangle^B\left\langle 1\right|, \tag{11}$$

where $\left|\varphi\right\rangle_A = \left|0\right\rangle + e^{i2\phi}\left|1\right\rangle$. Next, Gregory will perform measurements in both partitions of the probe and he will correlate the data in a suitable way in order to get the best precision in the phase estimation. If Gregory obtains the outcome 1 for the measurement on σ_z^B basis ($\{\left|0\right\rangle_B, \left|1\right\rangle_B\}$), he has to disregard such an event from his statistical sampling, since there is no information about the phase ϕ in this event. When he gets the outcome 0 for the measurement on the partition B (with probability $\mathcal{P}_{B=0}^G = 1-\frac{\eta}{2}$), he goes ahead and measures partition A adding this event to his statistics. In this case, Gregory will get the post-selected probability $\mathcal{P}_{A=0}^G = \frac{(1-\eta)(1+\cos(2\phi))}{2-\eta}$ for detecting the state $\left|0\right\rangle_A$. The fringe visibility for Gregory's coherent strategy is given by $\mathcal{V}^G = \frac{\max(\mathcal{P}_{A=0})-\min(\mathcal{P}_{A=0})}{\max(\mathcal{P}_{A=0})+\min(\mathcal{P}_{A=0})} = 2\frac{1-\eta}{2-\eta}$.

In the local adaptive strategy, after Bob's measurement (with probability $\mathcal{P}_{b=\pm}^B = \frac{1}{2}$) Alice has to perform a unitary conditioned to Bob's outcome (as described in Fig. 2(b)) over the state in Eq. (9). Alice detects the state $\left|0\right\rangle_A$ with probability $\mathcal{P}_{a=0}^A = \frac{1}{2}\left[1+(1-\eta)\cos(2\phi)\right]$. In this case, the fringe visibility of Alice's and Bob's adaptive strategy reads $\mathcal{V}^{AB} = 1-\eta$. Thus $\mathcal{V}^G > \mathcal{V}^{AB}$ for $\eta \neq 0, 1$. It is known that the visibility of an interferometer bounds the precision of phase estimation [14]. This example explicitly draw a picture of our observation concerning the usefulness of coherent interactions to effectively reduce noise in Gregory's read-out strategy.

It is worthwhile to note that such a result does not depend on the presence or not of entanglement or quantum discord. It is a consequence only of the quantum nature of the probe and the ability to perform coherent interactions. For a fully classically correlated probe, given by $\rho_0^{AB} = \sum_{ab} p_{ab}\left|a\right\rangle\left\langle a\right|\otimes\left|b\right\rangle\left\langle b\right|$ (where $\{\left|a\right\rangle\}$ and $\{\left|b\right\rangle\}$ are orthonormal basis in space ξ_A and ξ_B, respectively), we have $\mathcal{F}_G(\rho_\phi^{AB}) \neq \sum_b p_b\mathcal{F}(\rho^{A|b}) + \mathcal{F}(\rho^B)$, thus $\Delta\mathcal{F} \neq 0$. In this case, the coherent interaction in the read-out processes transfers local accessible information to a shared kind, similar to what is discussed in Ref. [9]. Here, the Gregory's advantage relies on the quantum nature of the probe, which can support some coherence after the second C-NOT gate in the read out (depicted in Fig. 2(a)). Although the coherent processing enables an enhancement over the local adaptive one, for classically correlated probe the possibility to beat the shot-noise limit remains unclear.

Multipartite states. An example similar to the above one can be carried out for a multipartite case. An acute noise reduction is obtained by using coherent interactions in the paradigmatic Mach-Zehnder interferometer employing a noise $N00N$-state probe [15]. In this case, we can represent the noise probe as $\rho_0^{N00N} = \frac{\eta}{2^{N+1}}\mathbb{1} + (1-\eta)\left|N00N\right\rangle\left\langle N00N\right|$ (with $\left|N00N\right\rangle = (\left|N,0\right\rangle_{AB} + \left|0,N\right\rangle_{AB})/\sqrt{2}$). The phase shift is now introduced by the unitary $U = e^{i\phi a^\dagger a}\otimes\mathbb{1}$ (where $a^\dagger a$ is the usual photon number operator for mode A) and the encoded probe state is $\rho_\phi^{N00N} = U\rho_0^{N00N}U^\dagger$. Gregory's Fisher information turns out to be $\mathcal{F}^{N00N}(AB) = \frac{2^N N^2(1-\eta)^2}{2^N-(2^N-1)\eta}$. In the adaptive strategy, the sequence of adaptive Bob's measurements on the space spanned by the N-quanta is equivalent to Alice applying N times the phase in her qubit, resulting in the conditional Fisher information $\mathcal{F}^{N00N}(A|B) = N^2(1-\eta)^2$. Local Bob's Fisher information is also null in this case. Thus

$$\Delta\mathcal{F}^{N00N} = \frac{\eta N^2(2^N-1)(1-\eta)^2}{2^N-(2^N-1)\eta}, \tag{12}$$

which is grater than zero for a noise probe ($\eta \neq 0, 1$). The gap between the performance of Gregory's coherent strategy and Alice's & Bob's adaptive strategies scales quadratically in the number of quanta N in the $N00N$ state for a fixed amount of noise η. Such a quadratically over-performance due to the coherent interactions in the noise context is really remarkable. It is the same level of enhancement obtained in the noiseless scenario that leads to quantum metrology protocols beating the shot-noise limit. Of course, depending on the amount noise in the probe state, we could not beat the shot-noise limit. On the other hand, considering the same amount of noise for a classical and a quantum probe, we can beat the precision of any classical protocol by using coherent interactions in the read out of the encoded probe.

Summarizing, we showed that different read-out strategies may result in an enhancement of the estimation precision in the general scenario of quantum metrology. Coherent interactions is proven to play a non-trivial role in the probe read-out stage of parameter estimation protocols. It is responsible for an important noise reduction

strategy. This fact has a deep and fundamental importance for any measurement protocol under the unavoidable presence of noise. Moreover, $\Delta\mathcal{F} > 0$ also implies that we can employ coherent interaction to over-perform a classical metrology protocol. Even in a very noise environment, we can harvest quantum advantage in a metrology task, reaching the same level of quadratic quantum-enhancement of the noiseless context.

We are grateful for the financial support from UFABC, CAPES, and FAPESP. This work was performed as part of the Brazilian National Institute of Science and Technology for Quantum Information (INCT-IQ). KM is supported by the John Templeton Foundation, the National Research Foundation, and the Ministry of Education of Singapore. KM thanks UFABC for their hospitality.

* kaonan.bueno@ufabc.edu.br
† kavan@quantumlah.org
‡ lucas@chibebe.org
§ serra@ufabc.edu.br

[1] H. Crameér, *Mathematical Methods of Statistics* (Princeton University, Princeton, 1946); C.R. Rao, *Linear Statistical Inference and its Applications* (Wiley, New York, 1973).
[2] S. Kullback, *Information Theory and Statistics* (Dover Publications,1997).
[3] R.A. Fisher, Phil. Trans. R. Soc. A **222**, 309 (1922); R.A. Fisher, Proc. Camb. Phil. Soc. **22**, 700 (1925).
[4] V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. Lett. **96**, 010401 (2006).
[5] V. Giovannetti, S. Lloyd, and L. Maccone, Science **306**, 1330 (2004).
[6] V. Giovannetti, S. Lloyd, and L. Maccone, Nature Photonics **5**, 222 (2011).
[7] K. Modi, H. Cable, M. Williamson, and V. Vedral, Phys. Rev. X **1**, 021022 (2011); B. M. Escher, R. L. de Matos Filho,and L. Davidovich, Nature Physics **7**, 406 (2011); B. M. Escher, R. L. de Matos Filho, and L. Davidovich, Braz. J. Phys. **41**, 229 (2011).
[8] S. L. Braunstein and C. M. Caves, Phys. Rev. Lett. **72**, 3439 (1994); S. L. Braunstein, C. Caves, and G. J. Milburn, Ann. Phys. New York **247**, 135 (1996); S. Luo, Lett. Math. Phys. **53**, 243 (2000).
[9] X.-M. Lu, S. Luo, and C. H. Oh., Rev. A **86**, 022342 (2012).
[10] L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001); H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
[11] L. C. Céleri, J. Maziero, and R. M. Serra, Int. J. Quant. Inf. **9**, 1837 (2011); K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, arXiv:1112.6238 (2011).
[12] K. Modi and M. Gu, arXiv:0902.0735 (2012); J. G. Filgueiras, R. S. Sarthour, A. M. S. Souza, I. S. Oliveira, R. M. Serra, and L. C. Céleri, arXiv:1208.0802.
[13] M. Gu, H. M. Chrzanowski, S. M. Assad, T. Symul, K. Modi, T. C. Ralph, V. Vedral, and P. Koy Lam, Nature Physics **8**, 671 (2012).
[14] Z. Y. Ou , Phys. Rev. Lett. **77**, 2352 (1996); F. W. Sun, B. H. Liu, Y. X. Gong, Y. F. Huang, Z. Y. Ou, and G. C. Guo, Euro. Phys. Lett. **82**, 24001 (2008).
[15] A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. **85**, 2733 (2000); H. Lee, P. Kok, and J. P. Dowling, J. Mod. Opt. **49**, 2325 (2002); M. W. Mitchell, J. S. Lundeen, and A. M. Steinberg, Nature **429**, 161 (2004); I. Afek, O. Ambar, and Y. Silberber, Science **328**, 879 (2010).